École des Ponts
ParisTech

Syllabus 2021

« PAM'S » PARIS APPLIED MATHS SEMESTER
@ Ecole des Ponts ParisTech

Introduction to functional analysis

[Antoine Levitt]

Objectives

The aim of this short course is to present basic elements of functional and applied analysis to the students.

It will start with the notion of completeness and the theory of Banach spaces, focusing on spaces of sequences and continuous functions; and then turn to the study of operators on Banach spaces, in particular continuous linear operators.

The lecture will end with an introduction to ordinary differential equations.

Modalities

> 3 sessions of 3h

> 1 ECTS

Assessment

Homework assignement

References

Dedicated lecture notes in English

Analysis and Scientific Computing

[Gabriel Stoltz]

Objectives

This lecture provides an introduction to various domains of 20th century analysis, with some emphasis on techniques allowing to better understand and solve the partial differential equations which are encountered in various scientific domains (transport equation, heat equation, Schrodinger equation, wave equation, etc).

On the theoretical side, the lectures introduce Lebesgue spaces, which build upon the theory of Lebesgue integrals and measure theory (whose aim is to generalize the notion of Riemann integrals); as well as the theory of distributions, which allows to give a meaning to very singular solutions to partial differential equations. Useful tools such as the Fourier transform are also presented.

Finally, numerical methods to actually solve partial differential equations are constructed, either through spectral approaches based on Fourier series, or by finite difference methods. In both cases, hands-on sessions allow the students to witness the methods at play.

Modalities

> 12 sessions of 2.5h, taught using «flipped classrooms» (the content of the course has to be learned at home beforehand, while sessions with the teachers are devoted to discussions on subtle points of the theory, and exercices in groups of 3-4 students). Weekly office hours.

> 3.5 ECTS

Assessment

Two exams (6 points each), periodic quizzes (4 points) and homeworks (4 points).

References

Dedicated lecture notes are provided and further references are given on the course's webpage

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Probability theory

[Aurélien Alfonsi]

Objectives

This course provides the essential knowledge in probability for undergraduate students. The fundamental notions (probability space, random variable, law of a random variable, expectation,...) as well as the usual probability laws with real and integer values are presented. Emphasis will be put on providing the tools to characterize and calculate probability laws. We will present the different notions of convergence in order to fully understand the statements of the two fundamental theorems that are the strong law of large numbers and the central limit theorem. Finally, we consider classical numerical methods to simulate random variables and give an introduction to Monte-Carlo methods.

- Space of probability, random variables, law, expectation. Link to the preparatory class program when the probability space is countable.
- Real random variables, usual probability laws with densities, probability laws neither discrete nor with density, calculation of the probability law by using test functions, characteristic function (link with Laplace and generating function).
- Random variables with vectorial values, dependency, Gaussian vectors, calculation of the law by the test functions (Jacobian), characteristic function.
- Presentation of the different convergence modes (in probability, almost sure, in law, L^p). Examples and counter-examples.
- Strong Law of Large Numbers and Central Limit Theorem. Construction of confidence intervals.
- Simulation of random variables. Inverse transform sampling. Box-Muller. Simulation of a Gaussian vector.
- Monte Carlo method. Interest of the method, curse of the dimension. Some examples, and variance reduction.

Modalities

- 2.5 ECTS
- 7 sessions of 3h; lecture and exercice sessions are all done in classroom format with no more than 30 students. Each teacher will suggest to the students (at least twice) to solve optional exercises for the next lecture. One or two self-study sessions (kind of office hours) are planned. They will make it possible to give priority to student support (questions/answers about the course and the exercises done in class) and possibly complementary exercises.

Assessment

- Final exam.
- Bonus points (between 0 and 3) for participation in class and written answers to optional exercises, taken into account only for students who have less than 10 on the examination (with grade capped to 10).

References

Benjamin Jourdain, Probability and Statistics, Ellipses, 2009.

A web page containing teaching materials and course information is maintained
http://cermics.enpc.fr/~alfonsi/Proba1A.html

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Introduction to Optimization

[Frédéric Meunier]

Objectives

The main objective of this course is to introduce the students to the classical methods for solving elementary problems of continuous optimization in finite dimension, in particular the KKT conditions and the simplex algorithm. The secondary objectives are to show the variety of application areas (other areas of mathematics, industrial engineering, economics, ...) and to give a first experience of modeling for decision problems.

Modalities

> Each session starts by a 50-minute lecture in the lecture hall and then continues with a tutorial of 2h15 in small groups.
> Two extra sessions: one to introduce a short project to be done during the course; one to prepare to the final exam
> 2.5 ECTS

Assessment

An exam (2/3 of the final mark) and a short project (1/3 of the final mark).

References

Lecture notes are provided.

Algorithmics and C++ Programming

[Pascal Monasse]

Objectives

Students are supposed to have followed an introductory course on programming in one of the following languages: C++ (ideally), C or Java. This course will provide some basic notions of algorithmics, that will be applied to the development of a project programmed in C++.

The first part introduces standard data structures, such as vector, list, stack, queue, priority queue, and discusses the complexity of their operations and the possible implementations. Some important principles of efficient algorithms will be exposed, such as divide and conquer and dynamic programming. They are applied in practical sessions including classical sorting algorithms, Fast Fourier Transform and fast marching for solving a class of partial differential equations. Applications include image processing (copy/paste between images, contrast enhancement) and computation of geodesics.

The second part is an assisted project developed by groups of 2 or 3 students. Many subjects are possible, for example strategic games, video games, computer graphics, image processing. Code versioning and sharing with git will be introduced and more generally usage of programming tools will be presented. The end result should be a quality C++ code implementing a working program, which will be presented in front of the whole class in a short oral and visual demonstration.

Modalities

> 6 sessions of 2.5h, including 4h of formal lectures the remaining time consists in practical sessions where C++ code must be completed and finished as homework.
> 6 sessions of assisted project development.
> 2 optional sessions of 2.5h on remainder of basics of C++ programming for students who need it
> 3.5 ECTS

Assessment

Practical session C++ code (4), written exam (1), project code and oral defense (1).

References

Lecture notes are provided.

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Operations Research and Optimization

[Axel Parmentier]

Objectives

Operations research is the discipline of applied mathematics which provides decision support tools. In the industry, decision makers have typically too many possible choices to consider each of them individually. Operations research gives solutions to find a good choice, or even the best choice. Example of applications include route choice, network design or scheduling. Operations research is a must-have in the toolbox of engineers who solve resource allocation problems. This is notably the case of those working in supply-chain, network industries, infrastructure management, finance or information technology architecture. Big data opened the door to a huge number of new applications, and there are many industries where operations research is still underexploited.

At the end of the course, students will master the fundamentals of operational research: the ability to identify a problem that can be addressed by operations research, to model it as a mathematical problem, to propose relevant solution algorithms, and to evaluate the relevance of algorithms and the solutions they return. To that purpose, they will master the main mathematical tools of operations research (Sessions 1 to 6) and their application to an industrial problem (Sessions 7 to 12).

Modalities

> 12 sessions of 2h45, among which 6 are in small groups, 4 in full group, 2 at home.

> 3 ECTS

Assessment

One project (2/5 of the final grade) and one final exam (3/5 of the final grade).

References

Dedicated lecture notes are provided.

Statistics and Data Analysis

[Julien Reygner]

Objectives

This course is an introduction to the practical methods of statistics and data analysis and their mathematical foundations. In mathematical statistics, the topics covered are parametric estimation, confidence intervals and hypothesis testing. In data analysis, principal component analysis, clustering, linear and logistic regression are addressed. An introduction to nonparametric statistics and the theory of optimal estimation is also proposed.

Modalities

> 12 sessions of 2h, which are organized as follows : 7 lectures, 3 computer sessions, 2 «flipped classroom» sessions where students have to give a 15 min talk describing a specific statistical method and its application on a relevant data base.

> 2 ECTS

Assessment

One final exam (15 points) and the flipped classroom presentation (5 points)

References

Dedicated lecture notes are provided and further references are given on the course's webpage.

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Statistical physics
[Francois Chevoir]

Objectives

This course provides an introduction to statistical physics, whose aim is to understand the microscopic origin of the macroscopic behavior of large assemblies of particles (states of matter: solid, liquid or gas), and provide quantitative predictions. This field, born with the development of thermodynamics during the second half of the nineteenth century, received a new impetus with the advent of quantum physics at the beginning of the twentieth century, and still develops successfully, for example to characterize phase transitions, soft matter or nanomaterials, and study complex systems beyond material sciences.

At the end of this course, students will have an understanding and practice of calculating entropy and associated thermodynamic quantities for simple physical systems at equilibrium, and then in the case of phase transitions and non-equilibrium situations, from model microscopic systems (perfect gas, polymer, two-state systems ...).

Modalities

> 6 sessions of 3h: 45 minutes lecture, 2h in smaller groups
> Students should have followed some lectures on Thermodynamics
> 2 ECTS

Assessment

> correction of the weekly exercise prepared by a group of students: 20%
> written exam with documents (about 30 minutes) at the beginning of session 4: 20%
> written exam with documents (about 2h30) on the whole program at the end of the course: 60%

References

Handout of the course distributed before the first session. A reference book in english will be provided, and links related to the course will be provided. Tutorial distributed at each session.

Projects

Objectives

The aim of these semester-long projects is to complement the student's training at École des Ponts Paristech, both on theoretical aspects and numerical ones. Possible topics include:

> mathematical and numerical analysis for models in quantum physics.
> variational methods in physics (Lagrangian and Hamiltonian dynamics, relativity theory, etc).
> control of dynamical systems.
> functional inequalities in probability theory.
> longtime properties of Hamiltonian dynamics and their discretization.
> nested and Multilevel Monte-Carlo methods for conditional expectations.
> statistical modeling of random phenomena in physics, biology, finance...
> Markov Chain Monte-Carlo method and applications in computer science or data analysis.
> uncertainty propagation and quantification in industrial applications.
> implement, optimize, test and compare an algorithm published in a scientific journal or in conference proceedings in the fields of image processing or computer vision (ideally, the code should be of sufficient quality to consider a submission to the Image Processing On Line journal, IPOL, http://www.ipol.im/, a journal dedicated to reproducible research in image processing where each article is accompanied by open source, peer-reviewed code and an online demonstration system running directly in the user's browser without any extension).
> solving an optimization problem motivated by a concrete application from industrial engineering or transportation (modeling, construction of an algorithm, experimentation).

Students are welcome to suggest topics to study, or directions they want to emphasize

Modalities

> regular meetings with a senior researcher and weekly meetings with a junior researcher
> 6.5 ECTS per project

Assessment

Written report and script of the code used for numerical simulations (if relevant).